

# *Modeling in aquatic environment*

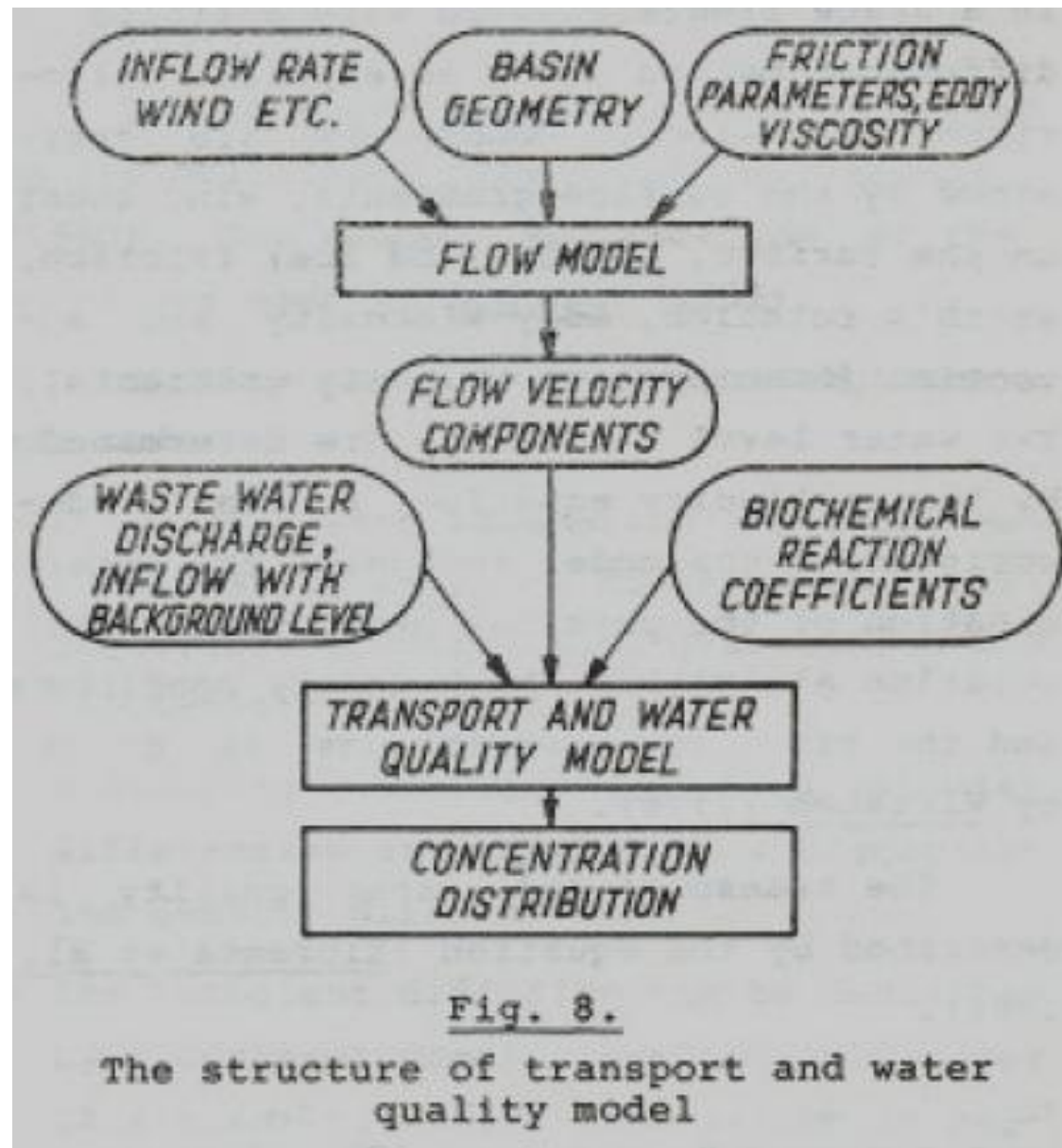
Lecture 8

Water quality models

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# Concentration equation in general form

$$\begin{aligned} \frac{\partial c}{\partial t} &= \frac{\partial qL}{\partial n} - u \frac{\partial c}{\partial x} - v \frac{\partial c}{\partial y} - (w - w_{s1}) \frac{\partial c}{\partial z} \\ &+ \frac{\partial c}{\partial x} \left( D_x \frac{\partial c}{\partial x} \right) + \frac{\partial c}{\partial y} \left( D_y \frac{\partial c}{\partial y} \right) + \frac{\partial c}{\partial z} \left( D_z \frac{\partial c}{\partial z} \right) \\ &+ R(T, c \dots) \end{aligned}$$

where,

- $c$  = concentration of simulated substance,  $qL$  = amount of loading release,  $n$  = length measure against release,  $u, v, w$  = advective velocity components in  $x$ -,  $y$ - ja  $z$ - directions,  $w_{s1}$  or  $w_s$  = settling velocity in the case of suspended solids, for soluble substances some value of  $w_{s1}$  is calibrated (!),  $D_x, D_y, D_z$  = dispersion coefficients,  $R(T, c)$  = biogeochemical changes in substance concentration

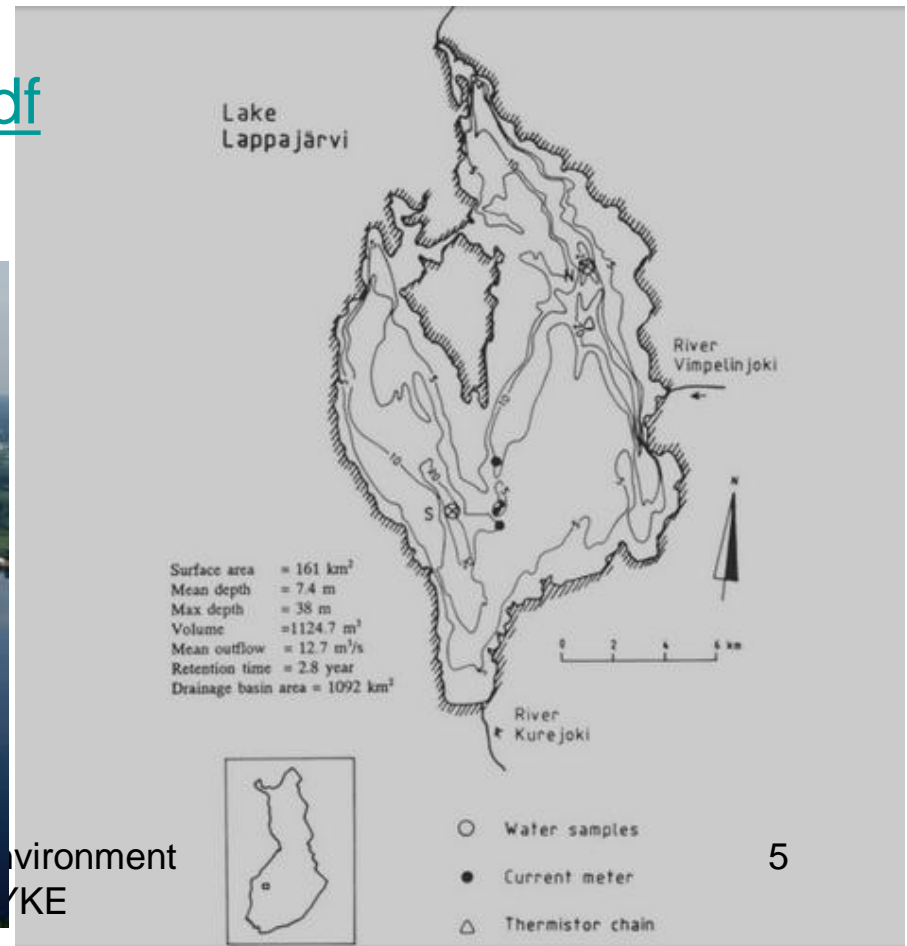
# Application of WQ-models

- We include :
  - Advection
  - Dispersion
  - Settling in water column and deposition on the bottom
- Bio- chemical processes
  - Decomposition, respiration, aeration, anaerobic release of P from the bottom
  - Select the most important variables concerning the problem
  - Oxygen, nutrients (like P,N), chlorophyll-a and some conservative substance (like Na)
  - Limiting factors (light, nutrients, ...) must be included
  - Temperature corrections must be included

# Lake Lappajärvi WQ-model

(Malve et al. 1991)

- PROBE temperature model
  - [Materials\Effects of Climate Change....pdf](#)
- PROBE-WQ model
  - [Materials\Lappajarvi\\_WQ.pdf](#)



# Oxygen model

Computation of dissolved oxygen Both abiotic and biotic factors affect the concentration of oxygen. The change of dissolved oxygen concentration as a function of time is described by the following equation:

$$\frac{dO_2}{dt} = \frac{K' \times \sqrt{W} \times (O_{2sat} - O_2) - K_1 \times BOD_t \times BRAT + \mu \times \alpha_1 \times CH - r \times \alpha_2 \times CH - \frac{SOD \times AREA}{V}}{V} \quad (7)$$

$K'$  = aeration constant =  $2.0 \cdot 10^{-4}$  cm/d,  $W$  = wind speed,  $z$  = layer thickness

$O_{2sat}$  = dissolved oxygen saturation concentration at the surface layer temperature

$O_2$  = dissolved oxygen concentration at the surface layer temperature

$K_1$  = BOD decay rate = 0.1 1/d (function of temperature,  $f(T)$ )

$BOD_t$  =  $BOD_7$  concentration

$BRAT$  =  $BOD/BOD_7$  = 1.5

$\alpha_1, \alpha_2$  = stoichiometric coefficients for growth and respiration = 0.1903

$\mu$  = growth rate of algae

$r$  = algal respiration coefficient = 0.065 1/d, ( $f(T)$ )

$CH$  = chlorophyll concentration

$SOD$  = bottom sediment oxygen demand, ( $f(T)$ )

$AREA$  = area of the bottom sediment

$V$  = volume of the water body

The first term on the right hand side describes aeration in the surface layer, the second one biological oxygen demand, the third and fourth ones phytoplankton growth and respiration, and the last one bottom sediment oxygen demand.

# Phytoplankton biomass and ToTP

Computation of phytoplankton biomass Chlorophyll-a concentration is used as a relative measure of phytoplankton biomass. The rate of change of phytoplankton biomass is expressed as [8]

$$\frac{dCH}{dt} = \mu \times CH - r \times CH - \frac{SED}{h} \times CH \quad (8)$$

Phosphorus cycle Description of the phosphorus cycle is quite simple. Total phosphorus concentration in the lake is affected by external loading, phosphorus sedimentation and release of phosphorus under **anaerobic** conditions.

The change of total phosphorus concentration as a function of time is described by the following equation [8]

$$\frac{dTOTP}{dt} = -\frac{SEDP}{h} \times (TOTP)^2 + LOAD + \frac{RELEASE}{AREA \times h} \quad (11)$$

SEDP = net phosphorus sedimentation coefficient = 0.002 (m/d)/(µg/l)

LOAD = external loading

RELEASE = rate of phosphorus release from the sediment under anaerobic conditions

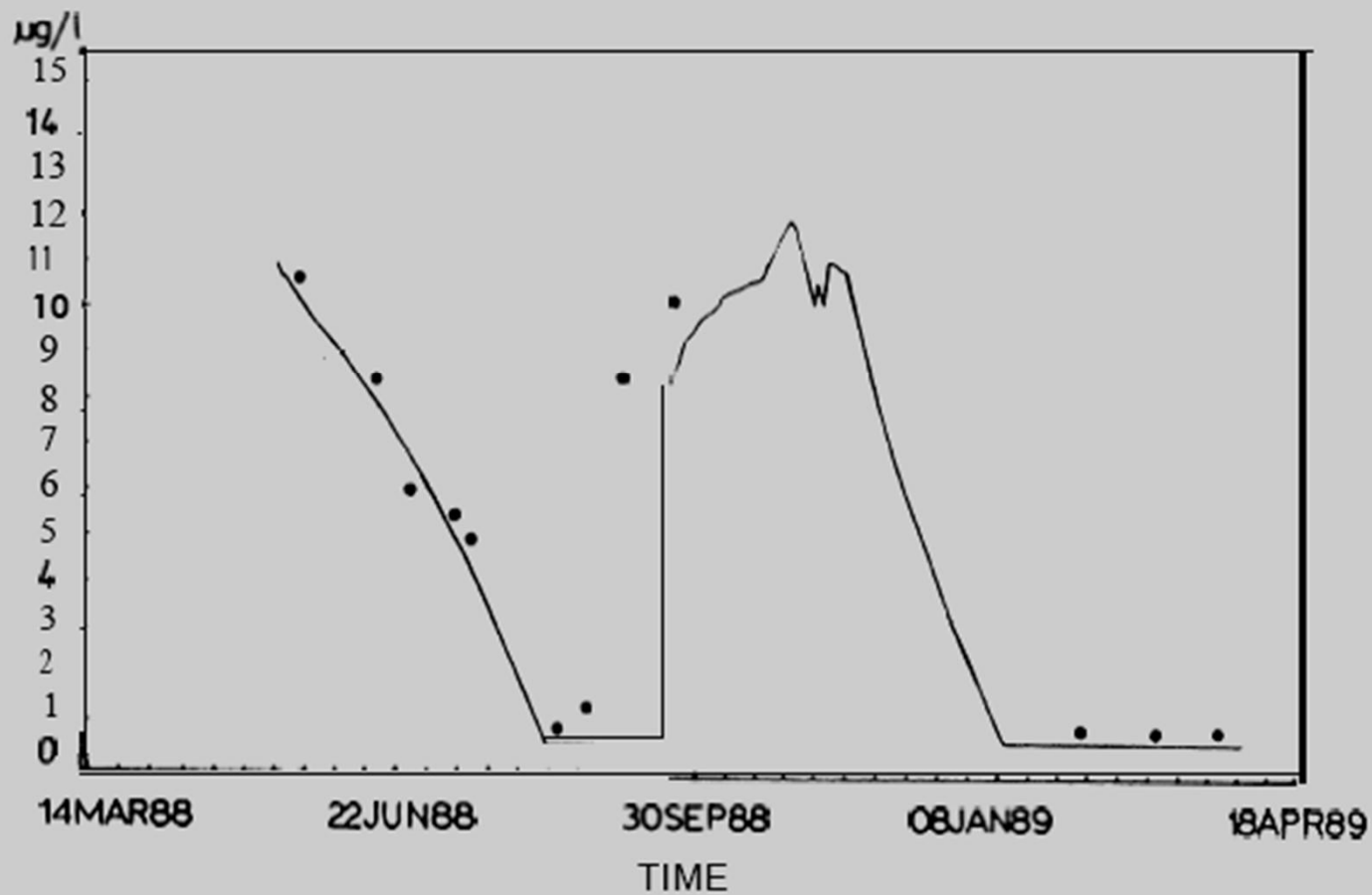


Figure 7. Observed (.) and calculated (-) oxygen concentrations in bottom layer (height 1 m). Calibration, summer 1988.



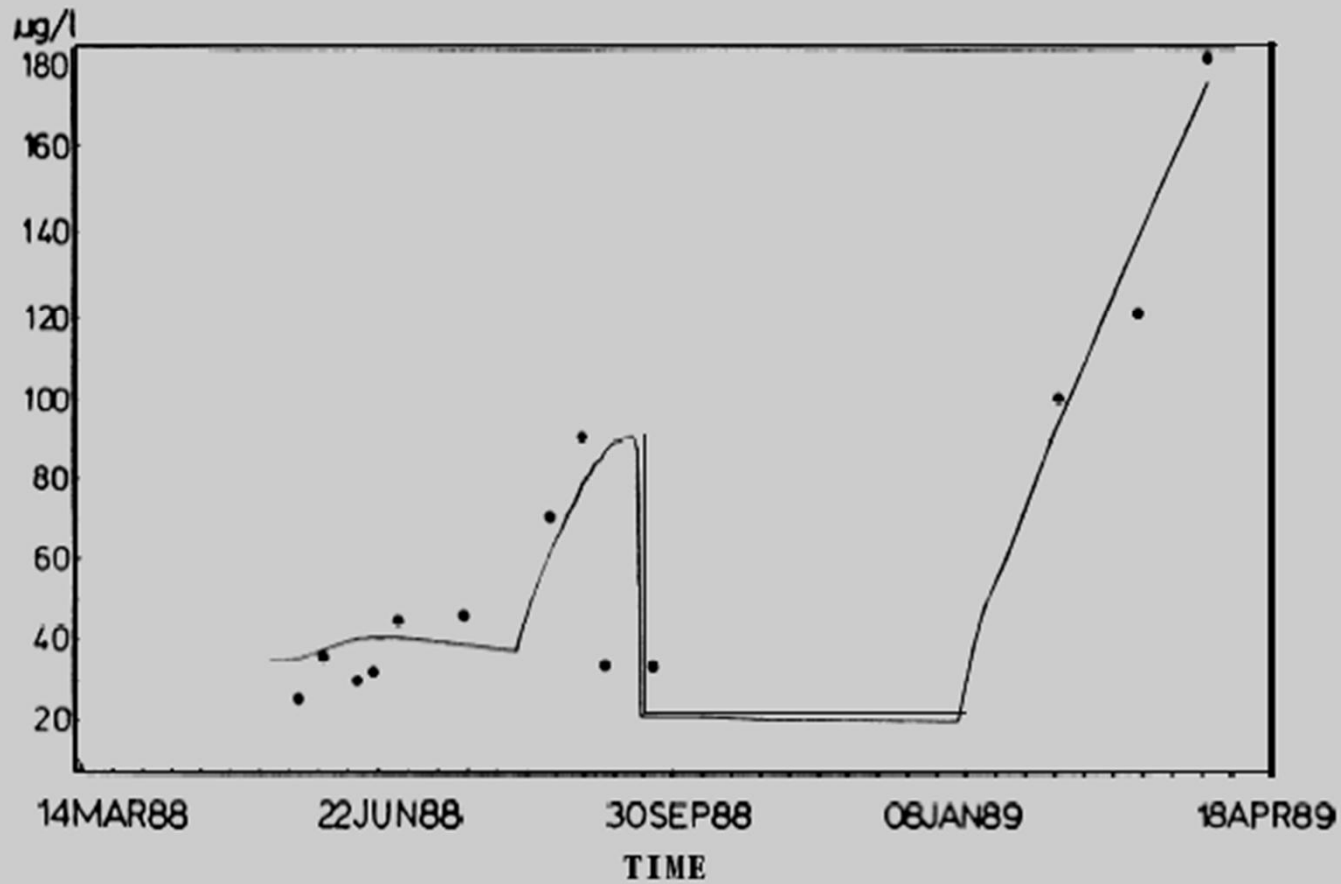


Figure 8. Observed (.) and calculated (-) total phosphorus concentrations in bottom layer (height 1 m). Calibration, summer 1988.

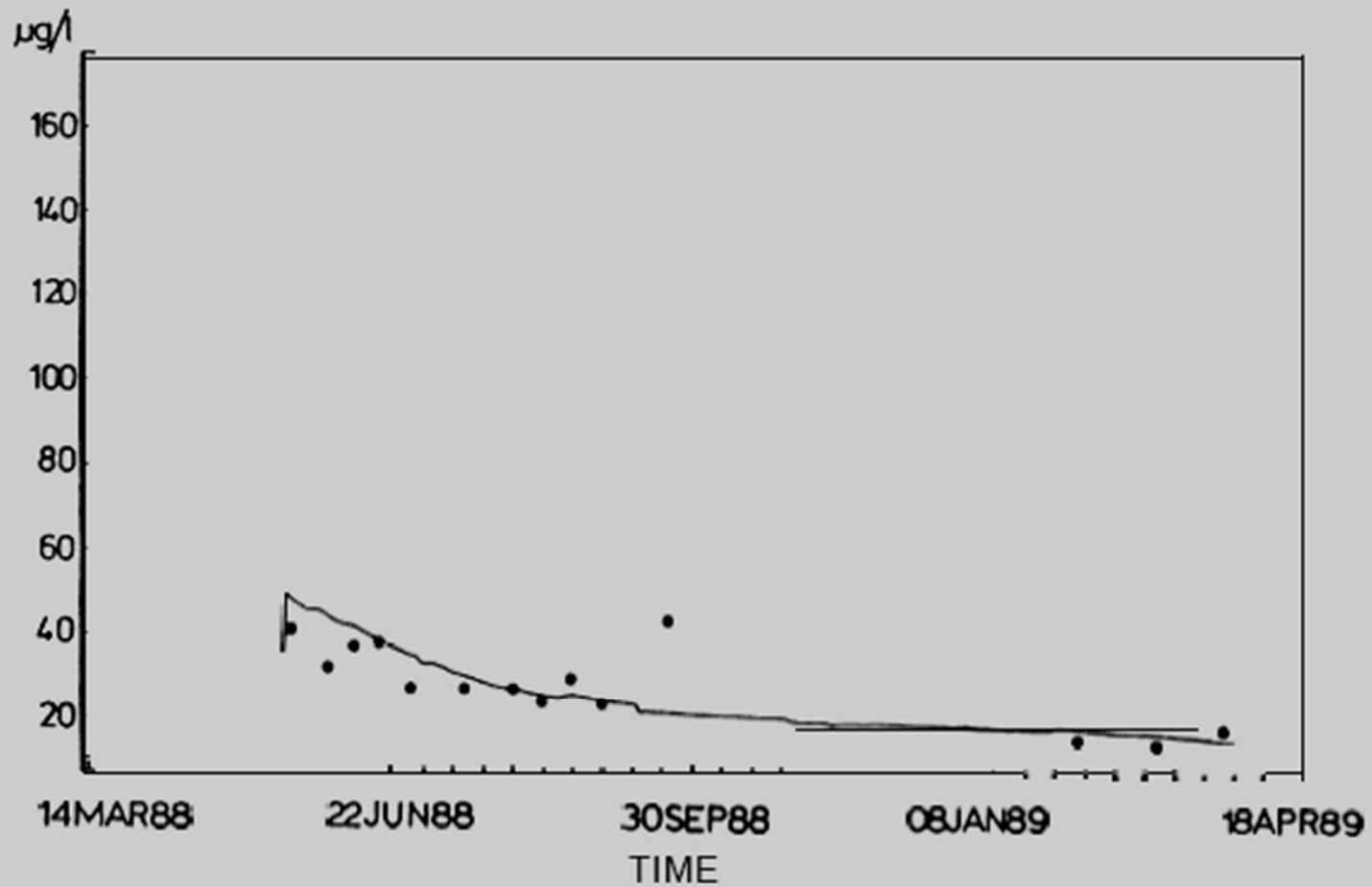


Figure 9. Observed (.) and calculated (-) total phosphorus concentrations in surface (0-1 m) layer. Calibration, summer 1988.

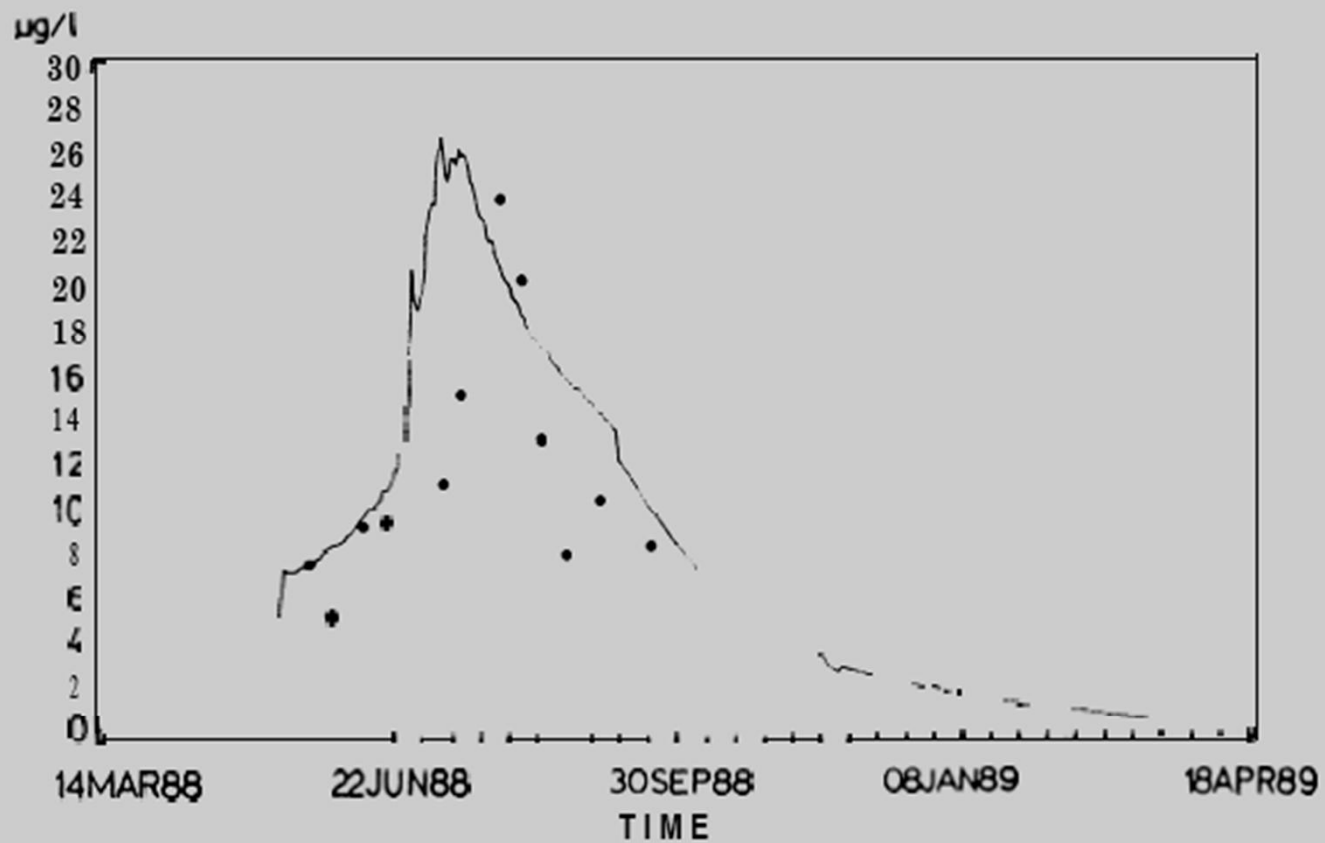


Figure 10. Observed (.) and calculated (-) chlorophyll-a concentrations in surface (0-1 m) layer. Calibration, summer 1988.

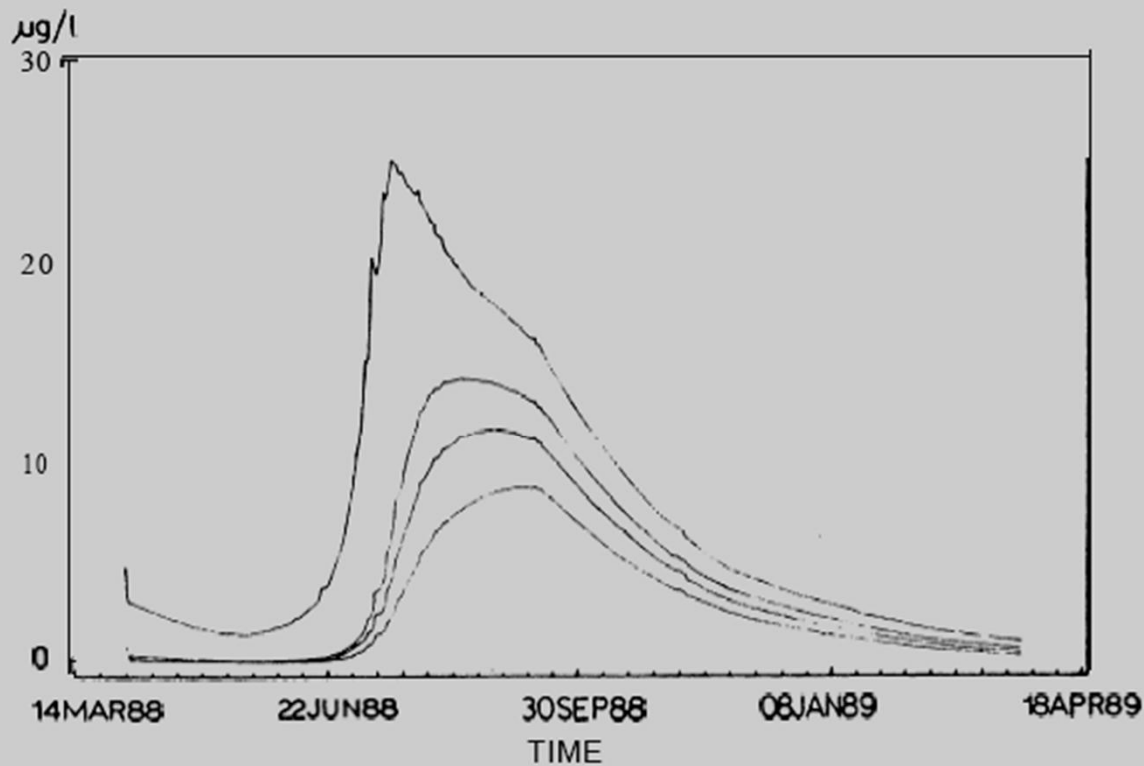
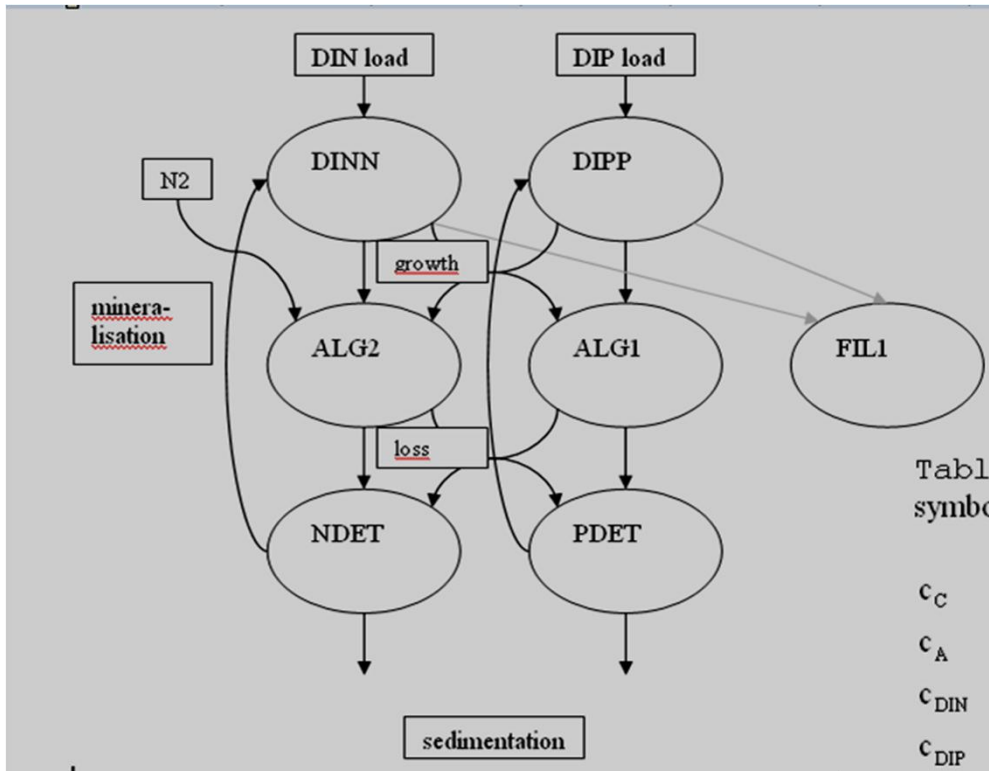


Fig. 11 Calculated chlorophyll-a concentration with loading levels 1 = 4.

1. Present total phosphorus loading	55 697 kg/year
2. Fast obtainable loading level	47 469 kg/year
-reduction of the present loading	14.8%
3. Desirable loading level	37 483 kg/year
-reduction of the present loading	32.7%
4. Loading level with the best available protection measures	30 689 kg/year
-reduction of the present loading	44.9%

# Interactions in EIA-SYKE-model



- DIP=dissolved inorganic P
- DIN=dissolved inorganic N
- Detritus= 'dead' particulate organic material in water

Table 1. Model variables.

symbol	definition	unit
$c_C$	Biomass of <u>cyanobacteria</u> (wet weight)	$g\ m^{-2}$
$c_A$	Biomass of the other algae (wet weight)	$g\ m^{-2}$
$c_{DIN}$	DIN concentration	$mg\ m^{-3}$
$c_{DIP}$	DIP concentration	$mg\ m^{-3}$
$c_{Ndet}$	Detritus nitrogen	$mg\ m^{-3}$
$c_{Pdet}$	Detritus phosphorus	$mg\ m^{-3}$
$L_{DIN}$	DIN load	$mg\ m^{-3}\ d^{-1}$
$L_{DIP}$	DIP load	$mg\ m^{-3}\ d^{-1}$
$I$	Total radiation	$MJ\ m^{-2}d^{-1}$
$T$	Temperature	$^{\circ}C$
$Ice$	Ice-cover (0,1)	-
$t$	Time	d
$h$	Depth of grid cell	m

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Table 2. Model parameters

Symbol	definition	reference	value	unit
$N_{inC}$	Nitrogen in <u>cyanobacteria</u>	Redfield, 1958	0.0193	-
$P_{inC}$	Phosphorus in <u>cyanobacteria</u>	Redfield, 1958	0.00268	-
$N_{inA}$	Nitrogen in the other algae	Redfield, 1958	0.0193	-
$P_{inA}$	Phosphorus in the other algae	Redfield, 1958	0.00268	-
$\mu_{Cmax}$	Maximal growth rate of <u>cyanobacteria</u>	calibration	0.5	d <sup>-1</sup>
$\mu_{Amax}$	Maximal growth rate of the other algae	Olli <i>et al.</i> , 1996	0.7	d <sup>-1</sup>
$R_{Cmax}$	Maximum loss rate of <u>cyanobacteria</u>	calibration	0.1	d <sup>-1</sup>
$R_{Amax}$	Maximum loss rate of the other algae	calibration	0.15	d <sup>-1</sup>
$K_{NC}$	Half-saturation coefficient of DIN for <u>cyanobacteria</u>	Tyrrell, 1999	0	mg m <sup>-3</sup>
$K_{PC}$	Half-saturation coefficient of DIP for <u>cyanobacteria</u>	Kononen & Leppänen, 1997	2	mg m <sup>-3</sup>
$K_{NA}$	Half-saturation coefficient of DIN for the other algae	calibration	7	mg m <sup>-3</sup>
$K_{PA}$	Half-saturation coefficient of DIP for the other algae	calibration	1	mg m <sup>-3</sup>
$K_{IC}$	Half saturation coefficient of radiation for <u>cyanobacteria</u>	calibration	20	MJ m <sup>-2</sup> d <sup>-1</sup>
$K_{IA}$	Half saturation coefficient of radiation for the other algae	calibration	15	MJ m <sup>-2</sup> d <sup>-1</sup>
$C_{min}$	Minimum biomass of <u>cyanobacteria</u>	calibration	0.5	g m <sup>-2</sup>
$A_{min}$	Minimum biomass of the other algae	calibration	0.01	g m <sup>-2</sup>
$A_{max}$	Maximum total biomass of algae	calibration	300	g m <sup>-2</sup>
$\beta_0$	Maximal detritus nitrogen mineralisation rate	Garber, 1984	0.018	d <sup>-1</sup>
$\gamma_0$	Maximal detritus phosphorus mineralisation rate	Garber, 1984	0.043	d <sup>-1</sup>
$\nu_{Ndet}$	Settling rate of detritus nitrogen	Heiskanen & Tallberg, 1999	1	m d <sup>-1</sup>

$v_{Ndet}$	Settling rate of detritus nitrogen	<u>Heiskanen &amp; Tallberg, 1999</u>	1	$m\ d^{-1}$
$v_{Pdet}$	Settling rate of detritus phosphorus	<u>Heiskanen &amp; Tallberg, 1999</u>	1	$m\ d^{-1}$
$S_{Ndet}$	Sedimentation rate of detritus nitrogen	calibration	0.16	$m\ d^{-1}$
$S_{Pdet}$	Sedimentation rate of detritus phosphorus	<u>Lehtoranta, 1998</u>	0.00	$m\ d^{-1}$
$T_{opt}$	Optimal temperature			
	for the growth of <u>cyanobacteria</u>	<u>Kononen &amp; Leppänen, 1997</u>	25	$^{\circ}C$
	for the growth of the other algae	calibration	15	$^{\circ}C$
	for losses	calibration	25	$^{\circ}C$
	for detritus nitrogen mineralisation	Garber, 1984	18	$^{\circ}C$
	for detritus phosphorus mineralisation	Garber, 1984	18	$^{\circ}C$
$a_T$	Coefficient for temperature limiting factor			
	for the growth of <u>cyanobacteria</u>	calibration	1.14	-
	for the growth of the other algae	calibration	1.001	-
	for losses	calibration	1.05	-
	for detritus nitrogen mineralisation	Garber, 1984	1.31	-
	for detritus phosphorus mineralisation	Garber, 1984	1.60	-
$I_{red}$	Radiation attenuation by ice	calibration	0.5	-
$h_{mix}$	Depth of mixing layer	calibration	20	m

Table 3. Model equations, rates and limiting factors.

Equations

$$\frac{\partial c_C}{\partial t} = (\mu_C - R_C)c_C \quad (1)$$

$$\frac{\partial c_A}{\partial t} = (\mu_A - R_A)c_A \quad (2)$$

$$\frac{\partial c_{DN}}{\partial t} = \beta c_{NDet} - \mu_A N_{inA} c_A h_{mix}^{-1} - \mu_C N_{inC} c_C h_{mix}^{-1} + L_{DN} \quad (3)$$

$$\frac{\partial c_{DP}}{\partial t} = \gamma c_{Pdet} - \mu_A P_{inA} c_A h_{mix}^{-1} - \mu_C P_{inC} c_C h_{mix}^{-1} + L_{DP} \quad (4)$$

$$\frac{\partial c_{Ndet}}{\partial t} = N_{inA} R_A c_A h_{mix}^{-1} + N_{inC} R_C c_C h_{mix}^{-1} - \beta c_{Ndet} - v_{Ndet} c_{Ndet} h^{-1} - S_{Ndet} c_{Ndet} h^{-1} \quad (5)$$

$$\frac{\partial c_{Pdet}}{\partial t} = P_{inA} R_A c_A h_{mix}^{-1} + P_{inC} R_C c_C h_{mix}^{-1} - \gamma c_{Pdet} - v_{Pdet} c_{Pdet} h^{-1} - S_{Pdet} c_{Pdet} h^{-1} \quad (6)$$

Rates

$$\mu_C = \mu_{Cmax} f_{CN}(c_{DN}, c_{DP}) f_{CI}(I) f(T) f_{AC}(c_A, c_C) \quad (7)$$

$$\mu_A = \mu_{Amax} f_{AN}(c_{DN}, c_{DP}) f_{AI}(I) f(T) f_{AC}(c_A, c_C) \quad (8)$$

$$R_C = R_{Cmax} f(T) (c_C - C_{min}) / c_C \quad (9)$$

$$R_A = R_{Amax} f(T) (c_A - A_{min}) / c_A \quad (10)$$

$$\beta = \beta_0 f(T) \quad (11)$$

$$\gamma = \gamma_0 f(T) \quad (12)$$



Limiting factors

$$f_{CN}(c_{DN}, c_{DP}) = \frac{c_{DN}}{c_{DN} + K_{NC}} \frac{c_{DP}}{c_{DP} + K_{PC}} \quad (13)$$

$$f_{AN}(c_{DN}, c_{DP}) = \frac{c_{DN}}{c_{DN} + K_{NA}} \frac{c_{DP}}{c_{DP} + K_{PA}} \quad (14)$$

$$f(T) = \exp \left[ \int_{T_{opt}}^T \ln \theta dT \right], \text{ where } \theta = a_T + (1 - a_T)T/T_{opt} \quad (15)$$

$$f_{CI}(I) = \frac{I(1 - l_{cel_{red}})}{I(1 - l_{cel_{red}}) + K_{IC}} \quad (16)$$

$$f_{AI}(I) = \frac{I(1 - l_{cel_{red}})}{I(1 - l_{cel_{red}}) + K_{IA}} \quad (17)$$

$$f_{AC}(c_A, c_C) = 1 - \frac{c_A + c_C}{A_{max}} \quad (18)$$

# Summary of WQ-model calculations

- Check that you have data to describe WQ in variable discharge and loading conditions
- Select those properties (variables), which describe best the effects of loading and concentrate calibration on them
- Use most simple parameterization of the variables
- First coefficient values from literature and by experience
- Compare the calculated and observed values of the selected variables
- Calibrate model tuning parameter values
- Select the conditions/scenarios (weather, discharge and loading) during which the effects are described ....and run the model!!

