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THE INFLUENCE OF SPATIAL WIND INHOMOGENEITY ON FLOW PATTERNS IN A SMALL LAKE

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Abstract—A two-dimensional vertically averaged flow model was applied to study the circulation patterns in Lake Belau (Northern Germany). The lake is north–south oriented, with a maximum extension of 2.2 km and is situated in a dead ice hole with steep slopes on the east and west coasts. A part of the coastal area is covered with forest, creating a strong shelter from prevailing south-westerly winds. The system of shallow water equations is discretized with the modified Utnes (1990) scheme which is characterized by a semi-decoupling algorithm. The continuity equation is rearranged to Helmholtz equation form. The upwinding Tabata (1977) method is used to approximate convective terms. Extensive wind (more than 60 observations) and current measurements (5 points) conducted all over the lake enabled us to verify the simulation results with observations. Under spatially homogeneous wind conditions the model predicts a two cell circulation system, that covers most part of the lake. Taking the spatial variation of wind speed due to shelter into account, the flow field changes drastically. The two cell system is replaced by one large cell, with a strong reverse jet along the western shore. The sheltering effect of the surrounding hills and vegetation have a pronounced effect on the circulation pattern. It appears that in general this fact cannot be neglected in numerical lake flow simulations. © 1999 Elsevier Science Ltd. All rights reserved

Key words—finite-element flow model, currents, sheltering effect, biological–hydrodynamic coupling

NOMENCLATURE

A	arbitrary mesh node,
B	boundary of a space domain,
E_A	upwind element of the node A,
f	Coriolis parameter,
g	acceleration due to gravity,
H	$\zeta + h$ = total water depth,
h	water depth below a horizontal datum,
k	wind resistance coefficient,
m	time layer index,
n	Manning roughness coefficient,
N	number of mesh nodes,
R, R^2	1D and 2D Euclid spaces,
S	arbitrary scalar function,
t	time,
U	upwind node for node A,
$V(u, v)$	velocity vector,
$W(W_x, W_y)$	wind velocity vector,
x, y	spatial Cartesian coordinates,
∇	gradient operator,
$\Omega \subset R^2$	arbitrary space domain,
τ	time step,
φ	global interpolation function,
ζ	water surface elevation above a horizontal datum,
ν	horizontal eddy viscosity.

INTRODUCTION

Wind induced currents play an important role in the dynamics of aquatic ecosystems. The transport of nutrients, pollutants and particulate matter (Serruya, 1975; Neilson and Stevens, 1987; Mossman and Roig, 1994), vertical mixing processes (Imboden *et al.*, 1983; Monismith, 1985; Reynolds, 1994), the distribution and resuspension of sediments (Bloesch, 1995), processes in makrophyte stands, reed zones and benthic systems (Losee and Wetzel, 1988; Ostendorp, 1989; Commito *et al.*, 1995) are largely determined or at least influenced by horizontal wind driven currents. The intensive investigations on the spatial heterogeneity of phytoplankton (Colebrook, 1960; Jones and Francis, 1982; Stauffer, 1982; Verhagen, 1994) and zooplankton (Lacroix and Lescher-Moutoue, 1995) during the last decades have also shown the close relationship between water flow and lake biology.

It follows that accurate diagnosis and prediction of lake currents are crucial for correct interpretation of limnological data and that the coupling of hydrodynamic and biological processes plays an important role in understanding of aquatic systems (Reynolds, 1989; Denman, 1994). Despite this, the effects of currents are still not taken into consideration in the interpretation of spatial patterns

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(Jakobsen and Johnson, 1987; Bast and Seitz, 1993). The effect of the littoral zone and shelter on currents is more pronounced in small lakes, which makes the model calibration more difficult.

In many applications the wind field over a lake is usually considered to be spatially homogeneous though the importance of spatial wind distribution is widely recognized. In the case of unsteady simulations the available time series of wind measurements are recalculated to near surface stresses using the logarithmic law of vertical wind speed variation (Henderson-Sellers, 1984). The necessity of considering the spatial variation of wind stresses was pointed out by Józsa (1990). The simulated currents in Keszthely Bay of Lake Balaton were in poor agreement with the observations when a conventional uniform wind shear pattern was applied. The best fit of simulated flow to observed pattern was achieved by empirically adjusting the spatial distribution of wind shear stresses starting with zero along the coast and increasing it linearly with a distance from the shore up to its full value in terms of the fetch length. The reasons for this considerable spatial wind variation are the surrounding topography, the sheltering effect of trees and buildings on the shore and the nonuniform lake surface roughness.

The aim of the present work is to emphasize the importance of the spatial wind irregularity to an accurate description of lake currents. The finite element solution of shallow water equations is described thoroughly. The proposed modification of the Utnes (1990) scheme possesses good stability properties and is suitable for long-term simulations. Detailed current and wind measurements in and over Lake Belau and their statistical processing are also presented and discussed.

GOVERNING EQUATIONS

The dynamics of flow in shallow lakes can be described with vertically integrated equations of motion and continuity (Weijan, 1992), given in vector form

$$\frac{\partial V}{\partial t} + (V \cdot \nabla)V + f \times V$$

$$= -g\nabla\varsigma + k|W|W - \frac{gn^2|V|V}{H^{4/3}} + \nu\Delta V, \quad (1)$$

$$\frac{\partial\varsigma}{\partial t} + \nabla \cdot (HV) = 0. \quad (2)$$

Here $V=(u, v)$ is a depth-averaged velocity vector, $H(x, y, t)=h(x, y)+\zeta(x, y, t)$ is the total water depth, ζ is the water surface elevation above a horizontal datum, h is the depth below datum, $W=(W_x, W_y)$ is the wind velocity vector, f is the Coriolis parameter, g is the acceleration due to gravity, n is the Manning roughness coefficient, k is the wind resistance coefficient and ν is the horizontal eddy viscosity.

The equations (1) and (2) belong to the class of incompletely parabolic systems (Gustafson and Sundström, 1978) when $\nu>0$. The boundary conditions are (Weijan, 1992):

$$\text{land boundary: } V|_{B_1} = 0 \quad (3)$$

$$\text{liquid boundary: } \varsigma|_{B_2} = \varsigma_B(t), \quad V|_{B_2} = V_B(t). \quad (4)$$

When $\nu=0$ the governing equations (1) and (2) constitute a system of quasilinear hyperbolic partial differential equations. In this case the non-slip boundary condition (3) is replaced with the slip one

$$V \cdot n|_{B_1} = 0, \quad (5)$$

where n is the unit vector normal to the boundary of the solution domain. Zero initial conditions are frequently used in practical applications to start the time integration

$$V = \zeta|_{t=0} = 0. \quad (6)$$

FINITE-ELEMENT SOLUTION

Using a time-splitting algorithm (Utnes, 1990) the momentum equation (1) is discretized as follows:

$$\frac{V^* - V^m}{\tau} + (V^m \cdot \nabla)V^* + f \times V^m$$

$$= k|W^{m+1}|W^{m+1} - \frac{gn^2|V^m|V^*}{H^{4/3}} + \nu\Delta V^*, \quad (7)$$

$$\frac{V^{m+1} - V^*}{\tau} = -g\nabla\varsigma^{m+1}. \quad (8)$$

When the time derivative is approximated with the forward difference the continuity equation takes the form

$$\varsigma^{m+1} = \varsigma^m - \tau\nabla \cdot HV^{m+1}. \quad (9)$$

Multiplying equation (8) by H, taking the divergence and substituting it in place of $\nabla \cdot HV^{m+1}$ into equation (9) the Helmholtz approximation of the semi-implicit continuity equation is obtained

$$\left[1 - \tau^2 g\nabla \cdot H\nabla\right]\varsigma^{m+1} = \varsigma^m - \tau\nabla \cdot HV^*. \quad (10)$$

The calculations are organized in the following way: an intermediate velocity V^* is calculated with equation (7), the water level elevation

$$\varsigma^{m+1}$$

is predicted with equation (10) and the corrected velocity V^{m+1} is obtained from equation (8).

The space domain Ω is divided into a sum of linear triangular elements. Unknown variables are approximated as a series of basis functions

$$V \approx \sum_{j=1}^{N} V_j \cdot \varphi_j,$$

$$\varsigma \approx \sum_{j=1}^{N} \varsigma_j \cdot \varphi_j, \tag{11}$$

where N is the number of mesh nodes and φ_j are the global basis functions. After substituting the decompositions equation (11) into equations (7), (8) and (10), multiplying according to the Galerkin method by the weighting functions φ_i^T, integrating over the space domain Ω and applying the Gauss theorem for the second-order terms the system of linear algebraic equations is derived

$$\begin{aligned}(M + \tau(\mathrm{CONV} + D + gn^2 F))V^* \\ = M(V^m - \tau(f \times V^m - k|W^{m+1}|W^{m+1})) \\ + \tau \int_B \varphi_i \nu \frac{\partial V}{\partial n} \mathrm{d}B,\end{aligned} \tag{12}$$

$$\begin{aligned}(M + g\tau^2 K)\varsigma^{m+1} = M\varsigma^m - \tau G(HV)^* + g\tau^2 \\ \int_B \varphi_i H \frac{\partial \varsigma}{\partial n} \mathrm{d}B,\end{aligned} \tag{13}$$

$$MV^{m+1} = MV^* - \tau g G \varsigma^{m+1}, \tag{14}$$

where the global matrices are expressed as follows:

$$\begin{aligned} M &= \int_\Omega \varphi_i \cdot \varphi_j^T \mathrm{d}\Omega, \\ D &= \int_\Omega \nu \nabla \varphi_i \cdot \nabla \varphi_j^T \mathrm{d}\Omega, \\ F &= \int_\Omega \varphi_i \cdot \varphi_j^T \cdot \frac{|V|^m}{H^{4/3}} \mathrm{d}\Omega, \\ G &= \int_\Omega \nabla \varphi_i \cdot \varphi_j^T \mathrm{d}\Omega, \\ K &= \int_\Omega \nabla \varphi_i \cdot H \nabla \varphi_j^T \mathrm{d}\Omega. \end{aligned} \tag{15}$$

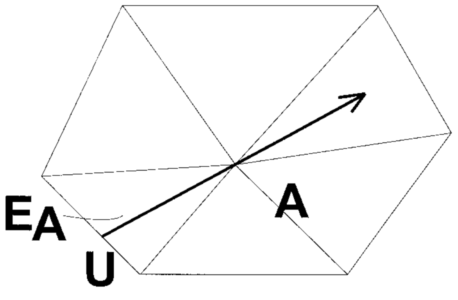

Fig. 1. A definition of the upwind element.

The CONV denotes the global convective matrix, modified according to the upwinding Tabata scheme (Tabata, 1977). This technique is a clear generalization of upwind finite differences for finite element method and can be briefly illustrated at an element level in the following way.

Let V: $\Omega \rightarrow R^2$ be a given velocity and S: $\Omega \rightarrow R$ be a scalar function. Consider a mesh node A. Let us define an upwind element E_A as such an element that A is a vertex of E_A and the line segment originating from A with a direction $-V(A)$ crosses E_A (Fig. 1). In the case of a piece-wise linear interpolation function $\nabla S(A)$ is a constant and the convection term can be approximated with a first order formula (Tabata, 1977)

$$(V \cdot \nabla)S(A) \approx |V(A)| \frac{|S_h(A) - S_h(U)|}{h}. \tag{16}$$

The systems of linear equations (12)–(14) are solved sequentially using a direct Gaussian elimination method (Weijan, 1992).

The discretization method presented above is different from the original Utnes (1990) scheme in two ways. First of all, the convection, diffusion and bottom friction terms are treated implicitly. This removes the stability limit imposed by the explicit treatment of the convection terms. No second order 'diffusion' term, following from the Taylor expansion of the convection term to balance the diffusion, is introduced. Instead, an intuitively clear, upwinding Tabata scheme is applied directly to the convection terms. Secondly, the same bilinear interpolation functions are used both for the velocity and water level, allowing for analytical calculation of integrals (15). This model was used earlier to study the water exchange in the northern part of Lake Ladoga (Podsetchine *et al.*, 1995), flow and sediment transport in Lake Karhijärvi (Podsetchine and Huttula, 1994) and in other applications.

CASE STUDY

Data collection

The main part of the Bornhöved Lakes in Schleswig–Holstein (Northern Germany) is a chain of four north–south oriented eutrophic lakes connected by the creek Alte Schwentine. One of the largest lakes is Lake Belau, with a maximum extension of 2.2 km, a surface area of 1.1 km^2 and an average depth of 9 m. Along the east and west coasts steep slopes rise to the glacial sediment plateaus, which are up to 22 m above the lake level (Fig. 2). The surroundings of Lake Belau are for the most part under intensive agricultural use. Only the slopes and some parts of the plateau on the west coast are covered with forest dominated by beech, fir and larch. The beech forest is nearly a hundred years old with a height of about 20 m. The other forested

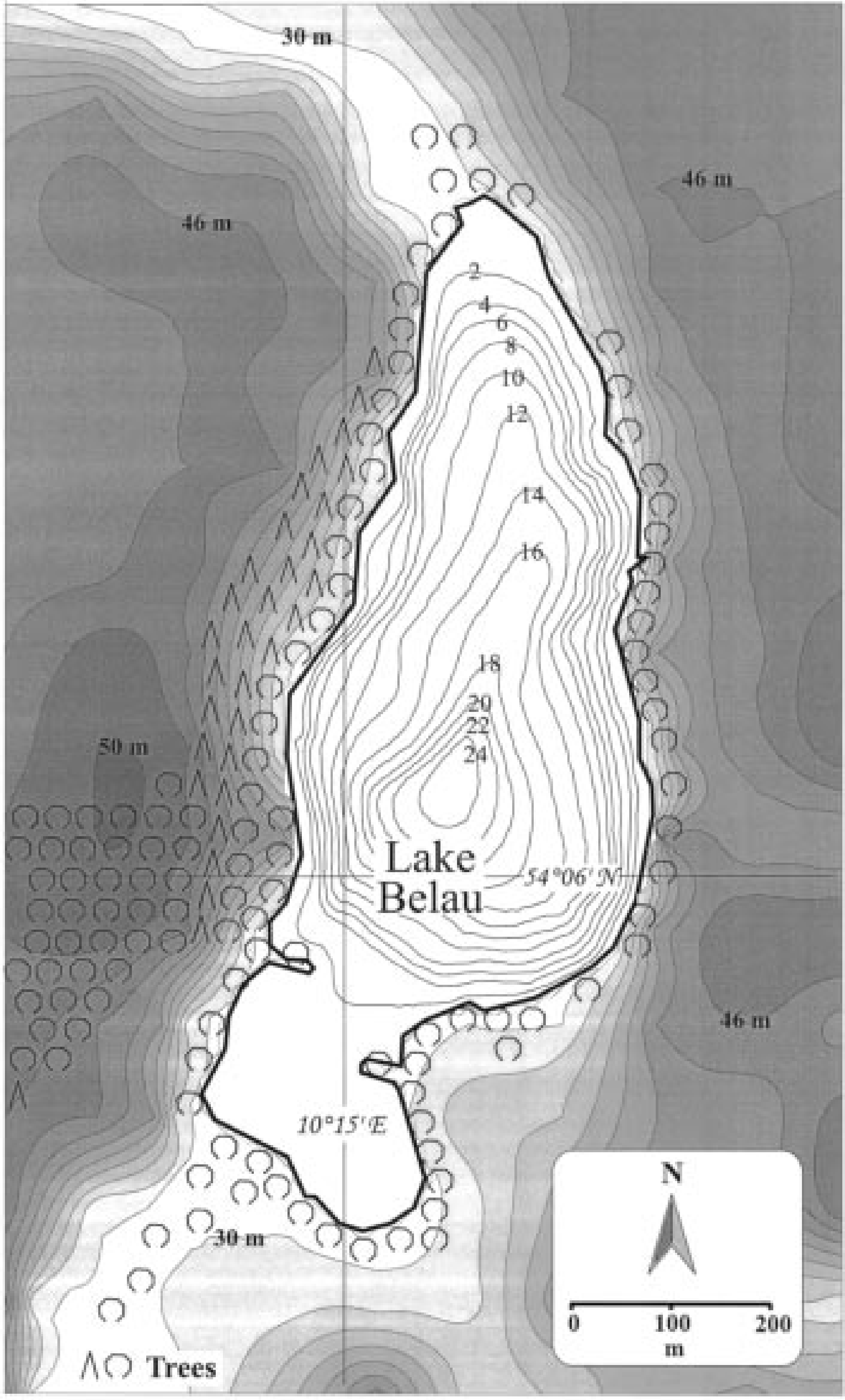

Fig. 2. A bathymetry (positive direction downward) of Lake Belau and a topography (positive direction upward) of its surroundings (North Germany).

areas and the narrow degenerated alder bogs along the larger part of the shoreline do not exceed a height of 10 m (Fig. 2).

All measurements on Lake Belau have been conducted with a small boat. During work it has been fixed in its position with two anchors. The position has been determined by using small buoys, GPS and was additionally estimated by using fixed objects on the shoreline. The time required to conduct measurements varied between 2 and 4 h. Flow velocities have been measured with the mobile inductive current meter ISM-2000 (MesSen Nord). Reliable current values down to some mm s^{-1} can be obtained when three minute averaging intervals are used. Currents were recorded in one meter to two meter vertical layers starting from a depth of 0.2 m down to the lake bottom.

For spatial measurements of wind speed and direction at a height of 1 m above lake a mobile anemometer of Thiessen Co. was used. Recorded wind speed was averaged over one minute interval. Additional wind data from a height of 1 m and with a time resolution of 10 min was provided by a floating station at the centre of Lake Belau. For long-term analysis of wind between 1990 and 1994, data from a meteorological station (a height of 16 m, 10 min recording interval) were used. The station is located about 200 m from the western shoreline of Lake Belau on the highest parts of the plateau on agricultural land. The values were aggregated to daily averages and supplied by E. Hollwurtel.

To get a detailed picture of the spatial wind structure over the lake surface and in order to calculate the shelter effects when south-westerly winds blow it was necessary to use all available wind measurements. For a wind direction of 220 degrees 6 recordings at 6 locations from July 27, 1997, 43 recordings at 5 locations from September 16, 1997 and 12 recordings at 12 locations from April 7, 1998 were used. The positions of all measurement points can be seen in Fig. 3.

Model application

The linear triangular mesh was created with the commercial software package ArgusONE version 3.0 (web site: www.argusint.com) and consisted of 516 nodes, 869 elements with the element size varying from 10 m in the south bay to 100 m in the central part of the lake. A time step of 15 s was used for the computations. The Manning roughness coefficient was set to $0.015\ m^{-1/3}$ s, the horizontal diffusion coefficient was equal to $0.01\ m^2\,s^{-1}$ and the Coriolis parameter was $1.176 \cdot 10^{-4}\ s^{-1}$. The zero normal flow condition was applied at the closed boundary instead of the slip boundary condition taking into consideration a small value of horizontal diffusion. This, otherwise would require a considerable refinement of the boundary layer and would be too time-consuming. Taking into account the small size of the lake, the integration period of 3 h was adequate to reach a steady state solution. Two wind scenarios were considered: spatially uniform south-westerly wind (a heading of 220°) with a speed of $6\ m\,s^{-1}$ and a spatially variable wind field obtained with bilinear interpolation of 67 measurement points. For processes affected by hydrodynamic, like transport, resuspension and sedimentation the knowledge of the flow under the south-westerly wind in Lake Belau is of utmost importance. That is why we concentrated our efforts on these specific wind conditions.

RESULTS

Most coastal parts of the lake are, to a certain extent, sheltered from wind by vegetation. The narrow north–south oriented valley with high ridges along the east and west coasts creates additional strong shelter from easterly and westerly winds. The forested plateau along the west coast, which rises up to 40 m above the lake level, affects most parts of the lake when westerly winds blow. On the other hand, winds coming from south to south–west and from north to north–west are channeled by the valley and can have strong effects on the lake.

Data over a 5-year period from 1990 to 1994 have been analyzed to investigate the probability of various wind directions in the Bornhöved lakes area. Only wind measurements at the height of 16 m above surface and 36 m above lake level were considered in an attempt to exclude topographical disturbances and effects of vegetation. Wind directions between NW and NE have a probability of 11%. South-westerly winds (180–270 degrees) have the highest probability of 42% in this area. On average there are about 50 d per year when the wind speed exceeds $6\ m\,s^{-1}$ at the height of 16 m. The share of south-westerly winds in these strong wind events is 63%. Not only do the south-westerly winds have the highest impact on the lake and the highest frequency but they are also related to high wind speeds more often than winds of other directions.

Calculated flow fields and their comparison with the observations are presented in Fig. 4. As expected under the uniform wind the flow follows the wind in shallow coastal areas and is directed against the wind in the central deep part of the lake, thus forming a two cell circulation system (Fig. 4(a)). Under the variable wind it is replaced by one large cell with a strong reverse jet along the western shore (Fig. 4(b)). This is in a good agreement with the measurements depicted in the same figure.

DISCUSSION AND CONCLUSIONS

The morphometrical properties of Lake Belau differ from those of the other 67 lakes in

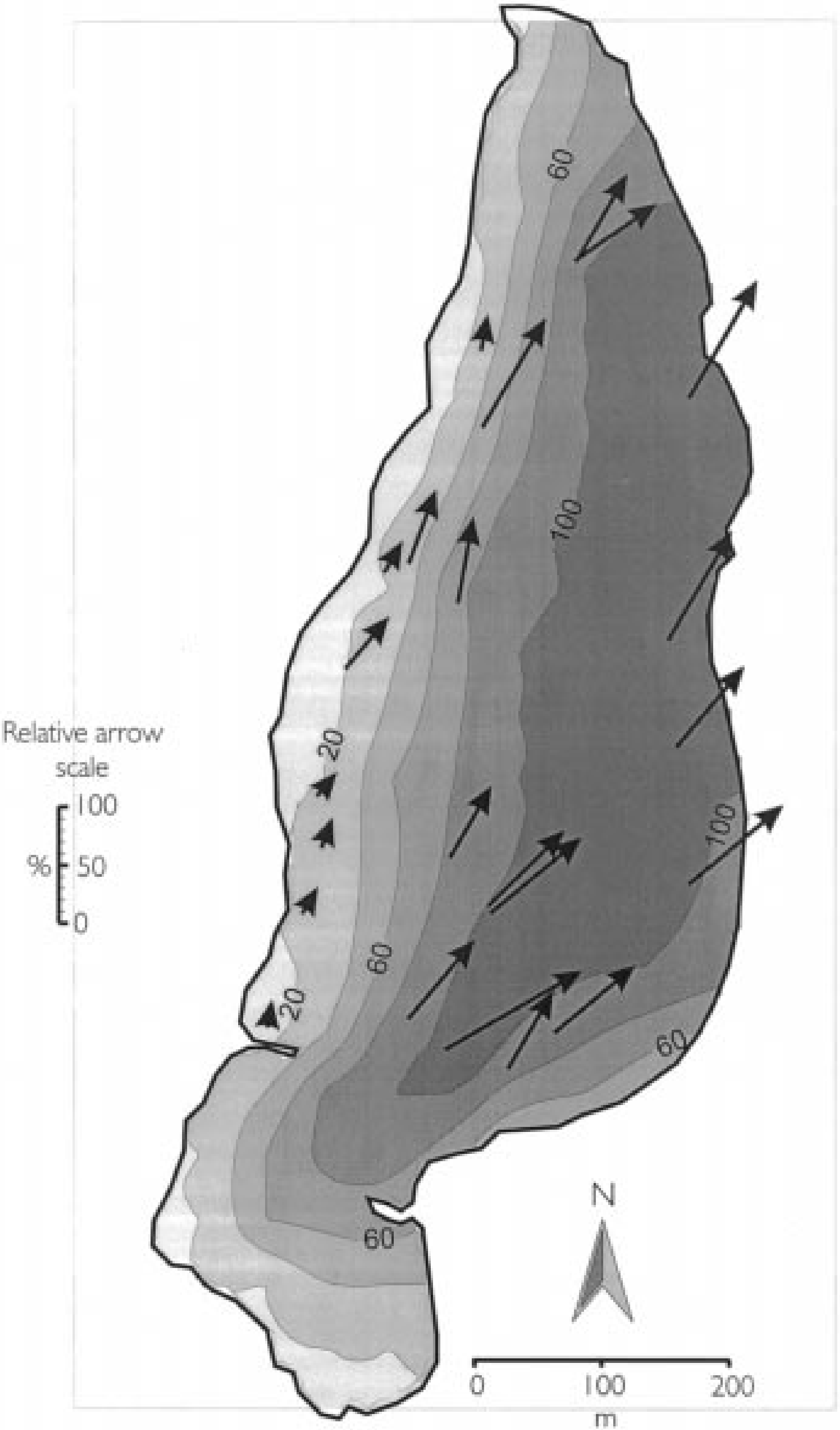

Fig. 3. Measured wind speed and direction as well as interpolated wind speed (contours in % of maximum value) for the model simulation. The measured data are a compilation of three sampling dates with prevailing wind from SW (April 7, July 27 and September 16, 1997) and average maximum wind speed of 6 m s^{-1}.

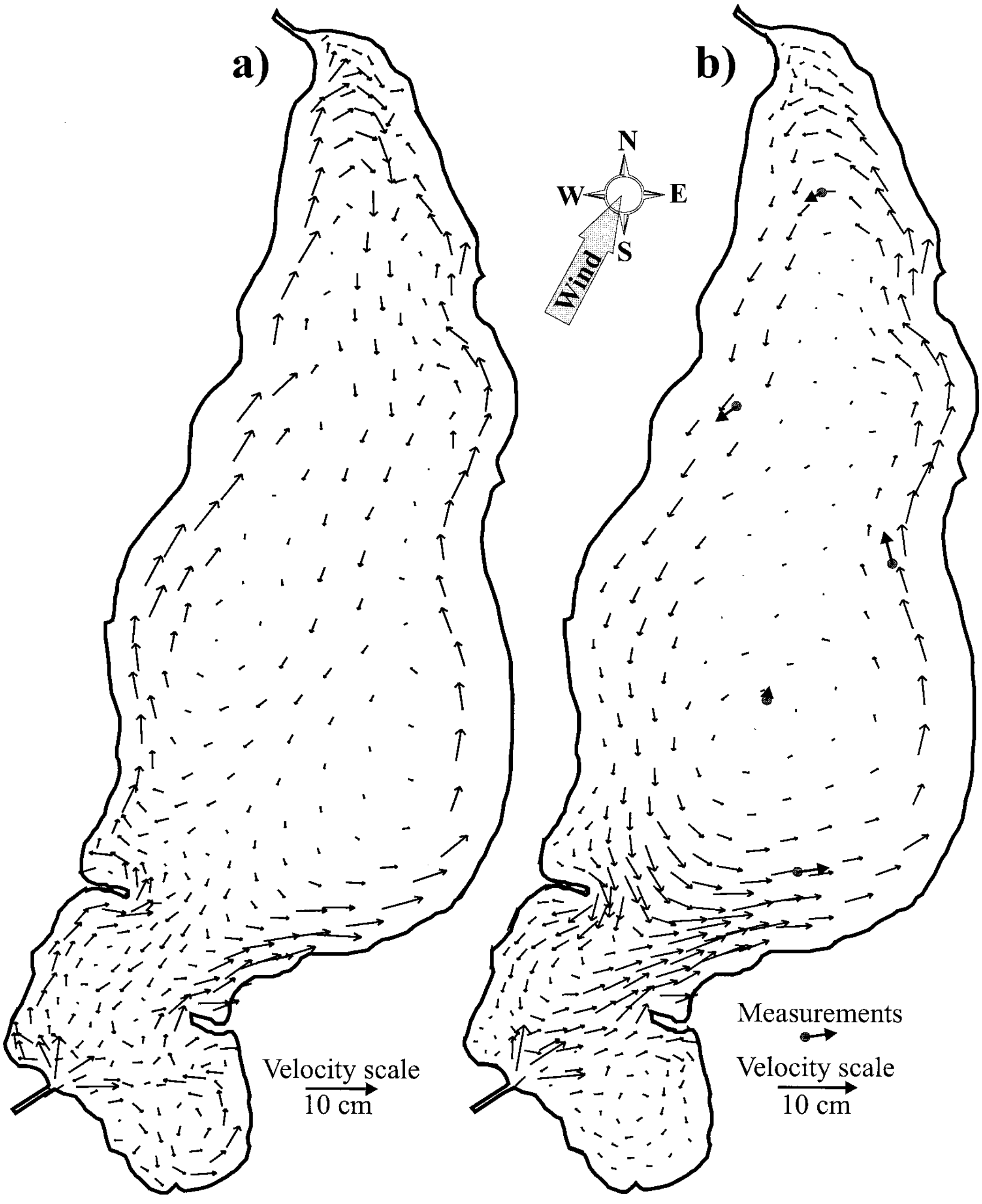

Fig. 4. Comparison of the simulated currents under the spatially constant (a) and variable (b) wind (a) with measurements.

Schleswig–Holstein. The relatively high maximum depth of 25 m as well as the high average depth of 9 m leads to the development of a strong and stable thermocline, which is not typical for North German lakes (Schernewski *et al.*, 1998a). The surface area of Lake Belau is 1.57 times larger than the median of 67 lakes in North Germany (0.7 km^2). The median value of the shore development index is equal to 1.6 compared to that of 1.5 in Lake Belau. For geographical conditions of North Germany, Lake Belau can thus be classified as a large lake with a typical basin shape. Most lakes in Schleswig–Holstein are located in the hilly area of glacial origin and have a genesis similar to that of Lake Belau. It is likely that most lakes in Schleswig–Holstein are characterized by a strong spatial variation of the wind field. The assumption that wind is spatially homogeneous could result in considerable errors of the flow field simulations.

The bottom roughness and wind drag coefficients vary within a wide range and are commonly used for model calibration (Henderson-Sellers, 1984). The reason for that is the complexity of the processes at air–water and water–bottom interfaces and insufficient amount of experimental data. Due to the small horizontal dimensions (order of 1 km) the fetch length in small lakes is limited and the influence of the surrounding topography on the wind field is greater than in big lakes. In addition to that, the part of the water surface under the shelter is larger in small lakes. Altogether these factors increase the error related to wind field approximation. The results also demonstrate that the spatial current and wind measurements with mobile instruments are possible and give reliable estimates of winds and currents. The spatial measurements of currents under steady state wind conditions provide more information about the horizontal circulation than sparse time-series at a few points because of the short transition periods in small and medium size lakes.

The models of Verhagen (1994) and Webster and Hutchinson (1994) clearly reflect that currents can be a dominating factor for phytoplankton patchiness in small lakes. Years ago, Colebrook (1960) stated that the passive drift with moving water is the main reason for the heterogeneous distribution of zooplankton in Lake Windermere as well. In addition to the observations of the phytoplankton patchiness (Stauffer, 1982; Camarero and Catalan, 1991) and horizontal zooplankton heterogeneity (Bast and Seitz, 1993; Lacroix and Lescher-Moutoue, 1995; Lair *et al*., 1996) covering the whole lake, the investigations of the horizontal heterogeneity of biological or hydrochemical parameters focused on small spatial scales have increased in number. The examples are the works of Jakobsen and Johnson (1987) and Lauridsen and Buenk (1996) in the littoral of small lakes. Most of these recent studies, especially on zooplankton, attempt to explain the observed spatial pattern exclusively by trophic interactions or by the strong swimming abilities of some species and neglect the possible effects of the water flow (Betsill and van den Avyle, 1994; Lair *et al*., 1996; Lauridsen and Buenk, 1996).

Current measurements and the simulations in Lake Belau show that the flow velocities in the littoral in front of the reed belt can reach values of several centimeters per second. Even in the reed belt, horizontal currents exceeding $1\,\mathrm{cm\,s^{-1}}$ are no exception (Schernewski *et al*., 1998b,c). These currents are above the swimming speed of even larger zooplankton. For example, during their diel vertical migration in Lake Constance the *Daphnia hyalina* moves with an average speed of about $0.08\,\mathrm{cm\,s^{-1}}$ (Stich and Lampert, 1981). There is no doubt that this is far below the maximum speed of this species, but it shows that even in small sheltered lakes currents can have considerable potential effects on the spatial distribution not only of phytoplankton, but of zooplankton as well. Therefore, flow models, with the ability to describe the flow pattern in sheltered and small lakes should become a commonly accepted tool in biological investigations.

The necessity to interpret small scale variations of biological parameters requires high resolution hydrodynamic models, which could simulate flow in small lakes with high accuracy. For most applications in the shallow littoral two-dimensional flow models are sufficient, but they should take wind shelter and the effects of the littoral, like the increased roughness in reed belts, properly into account. The modern computational facilities allow for high resolution flow calculations with the spatial scale of biological studies. At fine scales the more accurate description of subgrid scale processes requires special attention. More complex turbulent closure schemes instead of a constant eddy–viscosity model, such as the $\varkappa$–ε turbulence model (Younis and Chaudhry, 1994) could be a suitable choice.

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REFERENCES

Bast S. and Seitz A. (1993) Differential horizontal distribution during a day–night vertical migration of some cladocerans in a hypertrophic lake. *Ergeb. Limnol. Beih. Arch. Hydrobiol.* **39**, 187–198.

Betsill R. K. and van den Avyle M. J. (1994) Spatial heterogeneity of reservoir zooplankton: a matter of timing? *Hydrobiologia* **277**(1), 63–70.

Bloesch J. (1995) Mechanisms, measurement and importance of sediment resuspension in lakes. *Marine Freshwater Res.* **46**(1), 295–304.

Camarero L. and Catalan J. (1991) Horizontal heterogeneity of phytoplankton in a small high mountain lake. *Verh. Internat. Verein. Theor. Angew. Limnol.* **24**, 1005–1010.

Colebrook J. M. (1960) Plankton and water movements in Windermere. *The Journal of Animal Ecology* **29**, 217–240.

Commito J. A., Thrush S. F., Pridmore R. D., Hewitt J. E. and Cummings V. J. (1995) Dispersal dynamics in a wind-driven benthic system. *Limnol. Oceanogr.* **40**(8), 1513–1518.

Denman K. L. (1994) Scale-determining biological–physical interactions in oceanic food webs. In *Aquatic Ecology*, eds. P. S. Giller, A. G. Hildrew and D. G. Raffaelli, pp. 377–402. Blackwell Science, Oxford.

Gustafson B. and Sundström A. (1978) Incompletely parabolic problems in fluid dynamics. *SIAM J. Appl. Math.* **35**(2), 343–357.

Henderson-Sellers B. (1984) *Engineering Limnology*. Pitman Advanced Publishing, Boston.

Imboden D. M., Lemmin U., Joller T. and Schurter M. (1983) Mixing processes in lakes: mechanisms and ecological relevance. *Schweizerische Zeitschrift fur Hydrologie* **45**(1), 11–44.

Jakobsen P. and Johnson G. H. (1987) The influence of predation on horizontal distribution of zooplankton species. *Freshwater Biology* **17**, 501–507.

Jones R. I. and Francis R. C. (1982) Dispersion patterns of phytoplankton in lakes. *Hydrobiologia* **86**, 21–28.

Józsa J., Sarkkula J. and Tamsalu R. (1990) Calibration of modelled shallow lake flow using wind field modification. In *Proc., VII Int. Conf. on Computational Methods in Water Resources, Venice, Italy*. Computation Mechanics Publications, Southampton.

Lair N., Taleb H. and Reyes-Marchant P. (1996) Horizontal distribution of the rotifer plankton of Lake Aydat (France). *Aquatic Sciences* **58**(3), 253–268.

Lauridsen T. L. and Buenk I. (1996) Diel changes in the horizontal distribution of zooplankton in the littoral zone of two shallow eutrophic lakes. *Arch. Hydrobiol.* **137**(2), 161–176.

Lacroix G. and Lescher-Moutoue F. (1995) Spatial patterns of planktonic microcrustaceans in a small shallow lake. *Hydrobiologia* **300–301**, 205–217.

Losee R. F. and Wetzel R. G. (1988) Water movement within submersed littoral vegetation. *Verh. Internat. Verein. Theor. Angew. Limnol.* **23**, 62–66.

Monismith S. G. (1985) Wind-forced motions in stratified lakes and their effect on mixed-layer shear. *Limnol. Oceanogr.* **30**(4), 771–783.

Mossman D. J. and Roig L. C. (1994) Mixing in transport. *Water Environment Research* **66**(4), 477–489.

Neilson M. A. and Stevens R. J. J. (1987) Spatial heterogeneity of nutrients and organic matter in Lake Ontario. *Canadian Journal of Fisheries and Aquatic Sciences* **44**, 2192–2203.

Ostendorp W. (1989) 'Die-/Back' of reeds in Europe – a critical review of literature. *Aquatic Botany* **35**, 5–26.

Podsetchine V. and Huttula T. (1994) Modelling sedimentation and resuspension in lakes. *Water Pollution Research Journal of Canada* **29**(2/3), 309–342.

Podsetchine V., Huttula T. and Filatov N. (1995) Water exchange in the straights of northern Ladoga: results of field studies and numerical simulations. *Physics and Chemistry of the Earth* **20**(2), 207–213.

Reynolds C. S. (1989) Physical determinants of phytoplankton succession. In. In *Plankton Ecology*, ed. U. Sommer, pp. 9–56. Springer Verlag, Berlin.

Reynolds C. S. (1994) The role of fluid motion in the dynamics of phytoplankton in lakes and rivers. In. In *Aquatic Ecology*, eds. P. S. Giller, A. G. Hildrew and D. G. Raffaelli, pp. 141–188. Blackwell Science, Oxford.

Schernewski G., Schulz F. and Schulz U. (1998a) Use of numerical classification methods for lake monitoring. *Archives of Nature Conservation and Landscape Research* **37**, 1–34.

Schernewski G., Huttula T. and Podsetchine V. (1998b) Phytoplankton patchiness caused by current circulations: examples of a shallow bay in North Germany. *Aquatic Sciences*, submitted.

Schernewski G., Huttula T. and Podsetchine V. (1998c) Coastal minima of Ceratium concentrations in a small lake: a result of hydrodynamic processes? *Hydrobiologia*, submitted.

Serruya S. (1975) Wind, water temperature and motions in Lake Kinneret: general pattern. *Internat. Verein. Theor. Angew. Limnol.* **19**, 73–87.

Stauffer R. E. (1982) Wind stress effects on chlorophyll distribution in stratified eutrophic lakes. *Limnol. Oceanogr.* **27**(1), 66–74.

Stich H. B. and Lampert W. (1981) Predator evasion as an explanation of diurnal vertical migration by zooplankton. *Nature* **293**, 396–400.

Tabata M. (1977) A Finite element approximation corresponding to the upwinding finite differencing. *Mem. Numer. Math.* **4**, 46–63.

Utnes T. (1990) A finite element solution of the shallow-water wave equations. *Appl. Math. Modelling* **14**(1), 20–29.

Verhagen J. H. G. (1994) Modeling phytoplankton patchiness under the influence of wind-driven currents in lakes. *Limnol. Oceanogr.* **39**(7), 1551–1565.

Webster I. T. and Hutchinson P. A. (1994) Effect of wind on the distribution of Phytoplankton cells in lakes revisited. *Limnol. Oceanogr.* **39**(2), 365–373.

Weijan T. (1992) *Shallow Water Hydrodynamics*. Elsevier, Amsterdam.

Younis M. and Chaudhry M. H. (1994) A depth-averaged $\varkappa$–ε turbulence model for the computation of free surface flow. *Journal of Hydraulic Research* **32**(3), 415–444.